OFDM Channel Estimation by an Adaptive MMSE Estimator

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Abstract- In this paper, an extremely low-complexity adaptive infinite impulse response (IIR) filters that approximate minimum mean square error (MMSE) channel estimation in multi-band orthogonal frequency-division multiplexing (MB-OFDM) systems has been proposed. It has been showed that, how the packet error rate (PER) can be significantly improved over conventional zero-forcing (ZF) estimation without incurring a significant increase in computational complexity. Computational complexity and minimum- mean-square-error(MMSE) analysis are presented to evaluate the efficiency of the proposed algorithm.

Index Terms-Additive white Gaussian noise(AWGN)channel,MMSE(minimum-mean-square-error)Channel estimation,multi-band orthogonal frequency division multiplexing (MB-OFDM).equalization.

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1.INTRODUCTION

Channel estimation is an important factor in an orthogonal frequency division multiplexing (OFDM) system to know the channel state information (CSI) at the receiver [1]–[3].Pilot signals are commonly employed to facilitate channel estimation. However, pilot signals constitute extra overheads on both transmission power and available bandwidth. Intuitively, data signals, if known or partially known at the receiver, may have the same function as pilot signals. This motivates the research on iterative channel estimation and signal detection, in which detection feedbacks are used together with pilots in channel estimation [5]–[8]. Furthermore, the accuracy of channel estimation can be improved if certain channel statistics, such as the channel power delay profile (PDP) [4], is available.

The simplest means for OFDM channel estimation is a zero-forcing (ZF) approximation of N complex coefficients to rotate and scale each of the symbols with N subcarriers. To keep complexity low, many OFDM equalizers ignore the significant correlation between the subcarriers. This means that the magnitude of the additive white Gaussian noise (AWGN) that degrades the channel estimation is independent of the channel length. In other words, an impulsive flat-fading pure-AWGN channel needlessly suffers from the same channel estimation error as a highly frequency-selective channel. This means that the packet error rate (PER) is not optimally estimated in short channels.

To improve performance and make use of subcarrier correlation, a minimum mean square error (MMSE) estimator can be used. Since a direct MMSE estimation requires an N×N matrix multiplication[9],it is very expensive in high rate low-power systems like multi-band OFDM (MB-OFDM)[10], which is the first ultra-wideband (UWB) technology to obtain international standardization [11].The channel estimation techniques mentioned in this paper are applicable to almost any OFDM systems.

This paper gives the details to balance the quality and the complexity of OFDM channel estimation in the context of the MB-OFDM standard. A theoretical analysis in Section II is mentioned, where an upper bound on performance is obtained. Section III then develops ultra-low complexity approximately- MMSE estimation techniques. In Section IV, the complexity reduction to enable the estimation to be adaptive to instantaneous channel conditions has been described. The final PER is then analysed through Monte Carlo simulations in Section IV and the findings summarized in the conclusions of Section V.

2.MOTIVATION

A model representing an OFDM system can be given as-

$$y = Xh + n \quad \ldots\ldots\ldots(1)$$

where,'y' is the post-FFT received vector, X is a diagonal matrix containing the transmitted symbol constellations, 'h' is a complex channel attenuation vector and 'n' is a vector of independent and identically distributed complex, zero-mean, Gaussian noise variables with variance (σ_n^2).Equation(1) is entirely in the frequency-domain. Without any kind of loss, an assumption is made, that the channel is normalized such that $E\{|h_k|^2\} = 1$ and $E\{|X_{k,k}|^2\} = 1$.The receiver channel estimation is usually performed with the help of a known training sequence. This allows a ZF channel estimation to be easily obtained as-

$$h_{ZF} = X^{-1}y = h\,\check{n} \ldots\ldots\ldots\ldots(2)$$

where , $\check{n} = X^{-1}n$.As per the earlier assumption that E{|Xk,k|²} = 1, the variance of the AWGN denoted by 'ň' will remain σ_n^2.It is probably that that such kind of ZF estimation does not exploit the correlation between subcarriers and that the mean squared error (MSE) of the channel estimate will be $1/\sigma_n^2$.

To minimize the MSE, an optimal linear estimation [12] can be denoted as-

$$h = W\widehat{h_{ZF}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where,

$$W = R_{hh}(R_{hh} + \sigma_n^2 I)^{-1} \ldots\ldots\ldots (4)$$

*Throughout this paper, the following matrix notation conventions are adopted: [.]H denotes the Hermitian transpose; [.]−1 the matrix inverse column and [.]k,n the element of the kth row and nth.

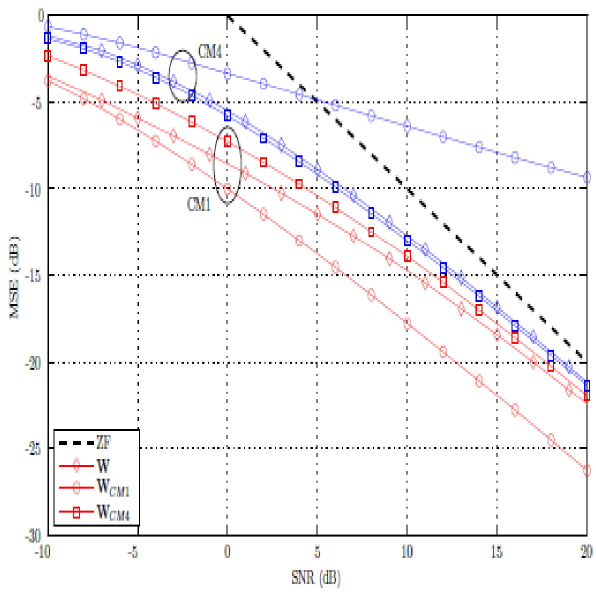

fig.1 MSE for different channel smoothing matrices W

with $R_{hh} = E\{hh^H\}$ denoting the auto-covariance matrix of the channel vector'h' and with 'I' denoting the N ×N identity matrix. So,equation (4) can be easily understood. When σ_n^2=0 and there is no AWGN, there is no need to exploit any subcarrier correlation and thus,W = I. When the AWGN increases such that $\sigma_n^2 >> 1$, we obtain, W = $1/\sigma_n^2 R_{hh}$ and subcarrier correlation is fully averaged to reduce the effect of the noise as much as possible.

Fig. 1 shows how critical the channel auto-covariance matrix R_{hh} is, which is used to calculate'W',that represents the channel conditions. The reference filter, denoted as 'W', is obtained from both CM1 and CM4 channel impulse responses(CIRs). The channel-specific filters, denoted as WCM1 and WCM4, use CIRs from only their respective channel model. It is observed that minimum MSE is achieved when the channel statistics closely matches the actual CIR. For example, if 'W' is based on an overestimation of the channel length, as occurs when WCM4 is used in CM1 channels, the correlation between subcarriers is under-utilized and the MSE increases by several dB relative to the reference W. Despite this, the performance is still superior than that of the original ZF channel estimate which allows to conclude that even such sub-optimal exploitation of subcarrier correlation is often better than the others. The losses that arises when 'W' underestimates the channel length, as in the case when WCM1 is used in CM4 channels, which are much more serious. Under such conditions, it is possible for the filter to degrade the ZF channel estimate. This is because, independent subcarriers are erroneously correlated. It can be thus concluded that, MMSE channel estimation is only suitable for extreme conditions that can exist in an OFDM system. In the case of MB-OFDM, where the CIRs can vary significantly depending on the distance between the transmitter and receiver, it may even be necessary to consider 'W' in real-time. For example, if an UWB transceiver pair is placed within a few centimetres of each other, then all CIRs will tend to be from the short CM1 channel. If 'W'is derived from all channel models that were used, the subcarrier correlation would be underestimated and the MSE would be suboptimal. Even when the statistics underlying 'W' are known accurately, there are two additional challenges that complicate practical implementation of MMSE channel estimation. First,
the filtering of the ZF estimate is computationally expensive. For example, direct implementation of equation (4) requires an expensive N × N matrix multiplication. Second, the IEEE channel models that underlay Fig. 1 are quite broad. This means that although the average delay spread of CM4 is greater than that of CM1, individual realizations of the CIR will vary considerably. The CM1/CM4 classification is therefore not appropriate for a practical receiver.

In the following two sections, several solutions to these problems are presented. First, in Section III, the complexity can be reduced sufficient for implementation under the severe constraints imposed on MB-OFDM hardware has been discussed. Then, in Section IV,a n adaptive algorithm that reuses calculations performed during packet synchronization has been calculated.

3. COMPLEXITY REDUCTION

Consider the complex baseband CIR,g = $F^H h$, where 'F' is the N ×N FFT matrix with $[F]_{k,n} = \frac{1}{\sqrt{N}} e^{-j2\pi kn/N}$. The time-domain channel auto-covariance matrix can be denoted as-

$$R_{gg} = E[gg^H] = F^H R_{hh} F \quad \ldots\ldots\ldots\ldots(5)$$

If it is assumed that each tap of 'g' has a uniformly distributed phase between 0 and 2π, as is the case in the IEEE UWB channel models, then Rgg will be a diagonal matrix. This forces both 'R_{hh} 'and 'W' to be circulant [4].

The MMSE can be denoted as the circular convolution of a ZF channel estimate with an N-tap finite impulse response (FIR) filter. The impulse response of this filter can be expressed as the equivalent to the first row of 'W', as w =[w0,w1, . . . ,wN−1]. The term 'impulse response' used here is with regard to the filter w; the fact that the filter is applied to a frequency-domain channel estimator which is irrelevant.

Direct filtering of the ZF channel estimator is not computationally easy since the circular convolution would require N^2 complex multiplications. One way to reduce this complexity is to use fast convolution [9]. Unfortunately, this approach involves two FFT/IFFTs: one to transform the initial frequency-domain ZF estimate into the time-domain and one to transform the smoothed MMSE estimate back into the frequency-domain. Given that each FFT/IFFT involves N/2 log_2N complex multiplications, the total complexity of a fast-convolution approach would be O(N log_2 N + N). Although this is a significant improvement over the O(N^2) complexity needed for direct circular convolution,1024 complex multiplications is still far too expensive for a MB-OFDM system where N = 128.

Fast-convolution can be simplified by truncating the time domain ZF estimate of the CIR to M taps. Although this avoids N−M complex multiplications, the continuing presence of an FFT/IFFT pair results in a still-too-high complexity of O(N log_2 N + 1).

In the same way the filter to M taps can be truncated while computing the fast convolution, it can also be truncated from w_n to P taps while using direct circular convolution. Indeed when P^2 < N 2 log_2 <N, that the filter w will be so short that it would be more expensive to perform the FFT and IFFT needed for fast convolution. In the context of MB-OFDM, this means that direct circular convolution is to be preferred over fast convolution if the filter w is truncated to P < 30 taps.

3.1 . Low-Complexity Filter Design

To further reduce the often prohibitive complexity of MMSE channel estimation, several low complexity alternatives has been designed. One of the MSE of a channel estimate can be given as-

$$\mathrm{MSE} = \frac{1}{N}\,\mathrm{Trace}(\mathrm{R_{ee}}) \quad \ldots\ldots\ldots\ldots\ldots(6)$$

Where,' Ree' is the auto-covariance matrix of the channel estimation error and is defined as-

$$R_{ee} = E\{(\hat{h}-h)(\hat{h}-h)^H\} = (R_{hh} + \sigma_n^2 I)W^H - R_{hh}W^H W R_{hh}{}^H + R_{hh} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

where 'W' denotes the smoothing matrix. If W = I, then the smoothing filter is unused and MSE = σ_n^2 since the ZF channel estimation is used as it is.It is observed that defining 'W' as per (4) gives the optimal MMSE[12].

Several low-complexity IIR filters that approximate 'W 'is described. This is done by minimizing (7) with the filter coefficients as the unknowns. Since Rhh is only obtainable via numeric methods, the minimization is done by an iterative search. Although this is computationally lengthy, the problem remains as it is, since all the filter banks are of very low order. For example, the most complex filter that is considered has only three independent variables. Although any non-linear search algorithm could be used, the simplex method [7] has been selected that provides high efficiency for low-dimension search spaces.

For each filter, a transfer function and an impulse response is provided. The impulse responses are expressed in the form a = [a0, a1, ..., aN−1] and are incorporated into (7) through the circulant matrix 'Wa' which has'a' as its first row.

i) First-order IIR: The simplest approach to channel estimation smoothing is to use a first-order infinite impulse response(IIR) filter with real coefficients. The transfer function of this filter is-

$$\mathrm{T_{Fa}(z) = B\frac{z}{z - A}} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

where 'A' is the coefficient ;controlling the rate of decay and 'B' is the gain. This filter's impulse response is-

$$\mathrm{\alpha_n = BA^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(9)$$

with both 'A' and 'B' constrained to positive real numbers since complex coefficients introduces unwanted phase rotations in the filtered output. For stability, constrain A < 1 has been considered.

ii) Second-order IIR*:* A second-order IIR filter with the transfer function is-

$$\mathrm{TF_b(z) = B\left(\frac{z}{z - A} + \frac{z}{z - C}\right)} \quad \ldots\ldots\ldots\ldots.(10)$$

where,'C' is an additional real coefficient constrained to C < 1for stability.The corresponding impulse response is given as-

$$b_n = B(A^n + C^N) \quad \ldots\ldots\ldots\ldots\ldots\ldots.(11)$$

iii) Symmetric IIR: The correlation between OFDM subcarriers is symmetric. It is therefore desirable to consider both higher and lower subcarriers when smoothing the ZF channel estimation. Since the first- and second-order IIR filters defined are not symmetric,so they exploit only half of the available correlation. This problem is solved by defining a symmetric variant of the first-order filter of (9) as-

$$\overleftrightarrow{a_n} = a_n + a_{N-n} \quad \dots\dots\dots\dots\dots\dots (12)$$

and similarly for $b_n^{\leftrightarrow}$ .In terms of hardware realization, a symmetric IIR filter can be easily implemented by adding the results from two independent IIR filters that each operates over the same input data in opposite directions.

iv) Product Power Play: The most costly part of digital filtering is multiplication.If filter coefficients,are flexible then multiplications by using a product power play (PPP) [8] to approximate each filter tap with the sum-and-difference of Q binary shifts can be avoided. In other words,$\alpha \approx \sum 2^{\alpha_1} \pm 2^{\alpha_2} \pm \cdots \dots . \pm 2^{\alpha_Q}$ where 'α' is a real-valued constant and α_1 throug α_Qh are integers. Another benefit of this approach is that the integer constants α_Q can be stored using very little memory. For example, in a receiver where the channel estimates are stored with 8-bits of precision, only log_2 8 = 3 bits are required for each variable shift α_Q if $0 < \alpha \leq 1$. This memory consumption can be reduced even further if some α_Q are fixed, as could be the case for coefficients with a small dynamic range. The impulse response of a PPP first-order IIR Filter is denoted as $b_Q^{\leftrightarrow}$=2, which denotes a $b^{\leftrightarrow}$ filter wherein each coefficient is approximated as the sum of two variable shifts.It is to be noted that the use of a PPP is to simplify an FIR filter 'W'.Unfortunately, the large number of taps in FIR filter means that this will lead to large high-latency adder-trees.

4. ADAPTIVE FILTERS

In previous work [9], it was recommended that an all purpose'W' be calculated using a channel auto-covariance matrix R_{hh} that is representative of all possible channel conditions. The SNR used to derive this generic 'W' should be relatively high as a low SNR will lead to excessive correlation between subcarriers and thereby increase the MSE for short CIRs. This was seen in Fig. 1 when a'W' derived under CM1 was used in CM4. In this paper,several low complexity IIR approximations to the optimal MMSE filter has been constructed. The preferred $b_Q^{\leftrightarrow}$=2 filter is fully defined by the six constants {A1,A2,B1,B2,C1,C2} that denote the PPP coefficients for (A,B,C). This filter is very small as it requires only 6 log_2 B bits, with 'B' denoting the bits of precision in the ZF channel estimate, of read-only memory (ROM). For example, in an MB-OFDM receiver with an 8 bit ADC, the entire IIR filter can be stored in as a 18 bits data.

The low ROM requirements to store several complimentary smoothing filters that are tuned for S SNR ranges and K classes of CIR can be exploited. A receiver that uses an adaptive MMSE channel estimation as per an algorithm similar to that of Alg. 1. This approach is only practical if steps 2 and 3 are low complexity. Many receivers already estimates SNR during synchronization or as part of the ZF channel estimation. When a predefine training sequence is used, the SNR is trivially calculated as-

$$SNR = \frac{|\sum t_0(n)+t_1(n)|^2}{|\sum t_0(n)-t_1(n)|^2} \dots\dots\dots\dots\dots (13)$$

where t0(n) and t1(n) are the nth received samples of the first and second repetitions of the training sequence t(n). It should be noted that many standards, including MB-OFDM, require all receivers to estimate SNR for use in link quality indication (LQI). There is therefore no added complexity in reusing existing SNR estimates to select an appropriate MMS channel estimation filter. Coherence bandwidth is an effective measure of subcarrier correlation and is inversely proportional to the channel root mean square (RMS) delay spread [10]. Although this makes RMS delay-spread an excellent classifier, it is not practically given that it can be calculated after the channel estimation has been made.Therefore,a much coarser metric of coherence bandwidth that defines the zero-crossing rate of the ZF channel estimate and denote as $\hat{t}$. $\hat{t}$ which is easily calculated by adding the exclusive-OR of the sign-bit of each tap in the ZF channel estimate has been thought about. For example, in an MB-OFDM system, this will produce an adder-tree with log_2 N = 8 levels. Since the inputs to this tree are only 1-bit wide, the final output will be 8-bits if full adders are used.It has been therefore, concluded that calculating $\hat{t}$ does not add significant incremental complexity. Having thus defined low-complexity quantitative estimates for both SNR and subcarrier correlation, we now consider the calculation of S×K MMSE channel estimation filters via Alg.2. As each of the S × K MMSE filters requires a non-linear optimization, this algorithm is computationally expensive and can only be performed off-line.By classifying CIR by SNR and subcarrier correlation, we can reduce MSE by matching the MMSE channel estimation filter to instantaneous channel conditions. Although both the SNR estimates and subcarrier correlation estimates are corrupted by AWGN, it is noted that the worst-case impact of poor classification is no improvement over no classification. For example, consider the case where $\hat{t}$ is totally corrupted and contains no useful information. The resultant categorization of CIR will be entirely random. The 'K' independent R_{hh} will therefore be equivalent. Now a case has been considered where $\hat{t}$ is only roughly proportional to channel delay spread. The 'long' and'short' channels will be grouped together and this will cause each category of R_{hh} to be unique.

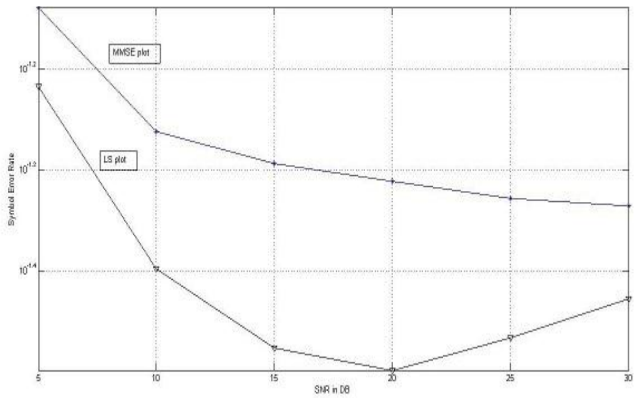

fig.2 Plot of SNR v/s SER for an OFDM system with MMSE/LS estimator based receiver

Fig. 2 shows how much the MMSE is reduced when adaptive filtering is used instead of using adaptive least square(LS) estiamator. In this simulation, the parameters are- S = 10 SNR categories and variable K = {1, 2, 4, 8} subcarrier correlation categories. CIRs were obtained randomly from CM1 through CM4.It has been observed that, there is negligible different in performance when K ≥ 4. Given that the low-order IIR filters can be stored with very few bits of ROM, the gains of adaptive filtering can be realized at a little cost.

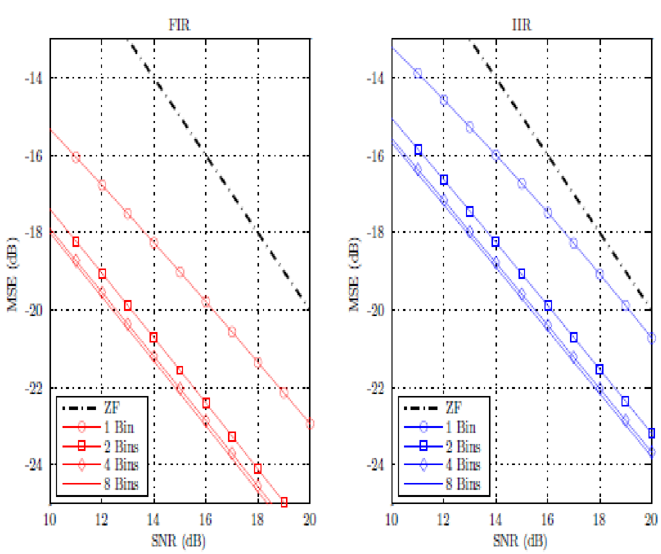

fig.3 MSE for adaptive filtering

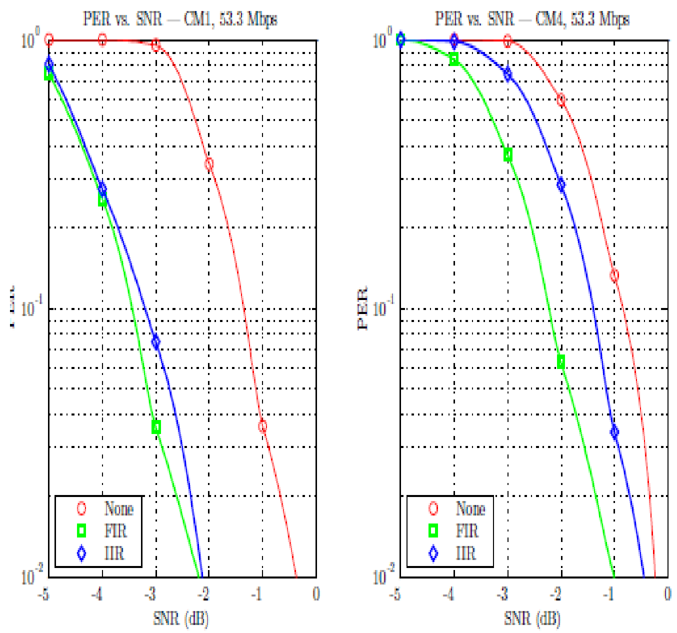

fig.4 PER for ZF and adaptive MMSE channel estimation at 53.3 Mbps

5. RESULTS

A Monte-Carlo simulation was used to obtain the impact of different approaches to MMSE channel estimation on the PER of an MB-OFDM receiver. The simulation environment implemented the complete MB-OFDM PHY [11] and considers forward error correction (FEC), time-frequency interleaving (TFI), time-domain spreading (TDS), frequency-domain spreading (FDS) and dual-carrier modulation (DCM).Also, no decision-feedback equalization (DFE) is used, which means that the channel estimate is based solely on the channel estimation sequence in the packet preamble. The IIR filters are adaptive to SNR, with S = 30, and channel length, with K = 4. The adaptive filters were derived using Alg. 2 and implemented using Alg. 1. The results of Fig. 3 and Fig.4 shows that there is very little PER difference between optimal FIR channel smoothing and a $b_Q^{\leftrightarrow} = 2$ IIR approximation. In most cases, the performance of IIR estimation smoothing is indistinguishable from that of the much higher complexity FIR estimation smoothing. The only time that FIR estimation smoothing is noticeably superior is in a highly frequency-selective CM4 channel at low SNR. Relative to ZF OFDM estimation,it can be concluded that IIR estimation smoothing offers significant PER improvement at nominal complexity in all channels.

6. CONCLUSION

In this paper, an extremely low-complexity IIR approximation to MMSE channel estimation has been carried out. In the context of MB-OFDM systems, it has been showed that an IIR filter can be used to achieve up to a 1.5 dB improvement in PER performance at a cost of less than 46 additions per subcarrier.

7.REFERENCES

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